

Modality, propositional functions, and logical learning: a historical look at the conflict between Kripke and Frege-Russell in the preface to Naming and Necessity

Modalidade, funções proposicionais e aprendizado lógico: um olhar histórico sobre o conflito entre Kripke e Frege-Russell no prefácio de nomeação e necessidade

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Abstract

We argue that the technical advantages of rigidity are not the most important factor in accepting Kripke's thesis against Frege-Russell (sense-wide-scope). Kripke's philosophical thesis is far removed from Frege-Russell's in that the former evaluates the function of "identity" in a different - metaphysical - way than the latter. In this paper, we argue that the discussion can evolve along a different path: evaluating and assessing the divergence between different conceptions of counterfactual knowledge, and studying the motives and prices that seemed reasonable to Russell-Frege to deal with the modal problem without expanding the extensional universe of classical logic. In particular, we will consider the merits of the Russellian option, understood as a theory of logical knowledge of the various ways in which a proposition can be false, depending on how we analyze the anti-extension of the proposition using propositional functions. Modal predicates can essentially be understood within the structures of fallible truth models by finding an extensional interpretation for counterfactual scenarios and extensionalizing complex identities. This provides a non-metaphysical understanding of what happens when one learns something logically about modality and counterfactuality.

Keywords: Modality. Rigidity. Logical fallibilism. Analysis. Propositional functions.

Resumo

Argumentamos que as vantagens técnicas da rigidez não são o fator mais importante na aceitação da tese de Kripke contra Frege-Russell (sense-wide-scope). A tese filosófica de Kripke está muito distante da de Frege-Russell no sentido de que o primeiro avalia a função da "identidade" de uma maneira diferente - metafísica - do que o último. Neste artigo, argumentamos que a discussão pode evoluir por um caminho diferente: avaliando e avaliando a divergência entre diferentes concepções de conhecimento contrafactual, e estudando os motivos e preços que pareciam razoáveis a Russell-Frege para lidar com o problema modal sem expandir o universo extensional da lógica clássica. Em particular, consideraremos os méritos da opção russelliana, entendida como uma teoria do conhecimento lógico das várias maneiras pelas quais uma proposição pode ser falsa, dependendo de como analisamos a anti-extensão da proposição usando funções proposicionais. Os predicados modais podem ser entendidos essencialmente dentro das estruturas de modelos de verdade falíveis, encontrando uma interpretação extensional para cenários contrafactuais e identidades complexas extensionalizantes. Isso fornece uma compreensão não metafísica do que acontece quando alguém aprende algo logicamente sobre modalidade e contrafactualidade.

Palavras-chave: Modalidade. Rigidez. Falibilismo lógico. Análise. Funções proposicionais.

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1 NAMING AND NECESSITY PREFACE: KRIPKE'S ARGUMENT ABOUT FREGE-RUSSELL'S INABILITY TO ACCOUNT FOR THE MODAL/COUNTERFACTUAL CONTENT OF SENTENCES

In the preface to *Naming and Necessity*, Saul Kripke criticizes Russell and Frege's failure to adequately semantically treat the counterfactual content of statements:

Consider: (I) Aristotle was fond of dogs. A proper understanding of this statement involves an understanding both of the (extensionally correct) conditions under which it is in fact true, and of the conditions under which a counterfactual course of history, resembling the actual course in some respects but not in others, would be correctly (partially) described by (I) (Kripke, 2001, p. 6).

Kripke believes that an adequate explanation of the meaning of a sentence models not only its extensional conditions. It also models its modal profile, by which we mean the counterfactual conditions that the sentence's chances resist (the possible scenarios in which its actual truth would not turn into a falsehood).

Consider the statement "The last pirate of the Caribbean islands was a butcher" According to Kripke, the truth or falsity of this sentence depends in some way on what it purports to specify in the universe of all possibilities. Thus, if this proposition is false, it can be opposed to any actual truth in an extensional and general sense. But it can also be false in a way that is specifically opposed to truth, as opposed to a possible conception of truth (a possible truth, to use Kant's term). Under these last conditions, the classical assumption of propositional bipolarity is an inconvenient obstacle that does not deserve unconditional respect: "a possible-world semantics involves, of itself, a considerable departure from Frege's own two value semantics" (Dummett, 1981, p. 576).

Kripke wants to recover the conditions so that the idea of knowing the "proposition" can be approached as a single problem: the problem of what kinds of cases or possibilities favor the assertion of that proposition. For this it is necessary that we can approach the propositional problem via the solution of another problem: the problem of rigidity, or the reference of the term in every possible world in which this reference exists. Kripke's argument supported the conclusion that the propositional problem as a whole, i.e., about the possible truth of a sentence, cannot even be adequately posed without posing the problem of

rigidity, i.e. how this sentence fix the reference to the possible worlds that support its assertion. We will deal with these theories in the next chapter.

To speak of 'the truth conditions' of a sentence such as (I) [Aristotle was found of dogs], it must be taken to express a single proposition-otherwise its truth conditions even with respect to the actual world are indeterminate (Kripke, 2001, p. 9).

This doctrine convincingly supplanted its competitor when it emerged, namely the theory of descriptions based on Frege's theory of meaning and Bertrand Russell's theory of incomplete symbols. However, it is not so clear that the merits of this victory are purely argumentative. The arguments of Frege and Russell have not been exhausted. But to better understand the context in which descriptivism appears as a reasonable option, it is necessary to understand the price these authors were willing to pay for their thesis, and why Kripke did not seem interested in the same debt.

2 RUSSELL'S AND FREGE'S SOLUTION REVISITED

Frege-Russell had a goal indeed, and it came at a price. In this context, the price seemed reasonable. The goal was: to explain the meaning of sentences as a function of their constituents. To do this, it was convenient that the functional components could be true or false in only one way. If there were different ways of being false, i.e., if a false sentence could be "read as true in a way specifically opposed to the facts," this would lead to a strange exception rule for the application of instances of the sentence. Micheal Dummett spoke in an apologetic tone in 1981 about this alleged restriction on classical theories, and showed how Frege could probably respond to Kripke's objections:

[...] Kripke's remarks in his preface give the appearance that he is contending that [Frege's semantics] forms only half of what goes to make up an understanding of the sentence and that there is another equally important ingredient. Frege did not draw the distinction drawn here between content and ingredient sense. He does not need to do so because he holds a subsidiary tenet: namely, that in a logically correct language, a constituent sentence will contribute to the determination of the truth-value of a complex one in which it occurs only via its own truth value. Given this subsidiary tenet, ingredient sense will coincide with content, and there will be no need to distinguish them (Dummett, 1981, p. 573).

It is necessary to recall some general characteristics of the noble loser, descriptivism. Descriptivism arises as a logical thesis to explain why some substitutions of co-referential expressions do not preserve truth value. One of these failed substitution problems was precisely the modal problem later used by Kripke as an argument against Frege's canon. It is important to remember, however, that the idea of substituting reference terms for descriptions was a deliberate solution to this problem. Descriptions can be seen as intensional objects or incomplete symbols. The first case was defended by Gottlob Frege. We refer here to an excerpt from the classic work of 1892, in the 1948 edition (*Sense and Reference*):

The expression "the least rapidly convergent series" has a sense; but it is known to have no referent, since for every given convergent series, another convergent, but less rapidly convergent, series can be found. In grasping a sense, one is not certainly assured of a referent (Frege, 1948, p. 211).

In the second case, the description is regarded as an incomplete symbol because it unlocks different semantic values in different propositional contexts. Russell's theory in *On Denoting* (1905) is an avowed attempt to model the meaning of general propositions - including existential propositions - that need to be better paraphrased in order to express their behavior according to the rules of logic (*Philosophy of logical atomism*):

Suppose you take such proposition as: "There is at least one thing in the world." That is a proposition that you can express in logical terms. It will mean, if you like, that the propositional function "x = x" is a possible one. That is a proposition, therefore, that you can express in logical terms; but you cannot know from logic whether it is true or false (Russell, 2010, p. 75).

Descriptions would account for this *logical knowledge available* when one isolates logical learning that is not extensional available. Descriptions give an extension to possible things. This is not a mere possible extension. It is a complete extension, but that is not subjected to Leibniz's law. However, one may make it amenable to that law. In order to recover extensionality for descriptive knowledge, one would have to work with refined logical paraphrases of sentence content. The most prominent solution to this problem was Russell's wide-

scopism. Wide-scopism is a thesis about the extensionalization of the modal and counterfactual profile of sentences. This thesis arises as a philosophical strategy to preserve the theory that the scope of a proposition can be generalized by a function that models its truth possibilities. When some anomalies seem to challenge this thesis, widescopism is a possible solution. The classic example of an anomaly in *On Denoting* is the sentence "The King of France is bald." Its ambiguous scope allows the sentence to be assigned both falsehood and necessary falsehood. This means that the second model of interpretation is incompatible with the first: if it is necessary false that the King of France is bald, it is false, and the truth tables in which there is a chance that the sentence is true are excluded.

The challenge is to reconcile these two interpretations as legitimate and not incompatible options. Russell solves the problem by logical analysis, decomposing it into different possible statements: "The King of France is not bald" is false if the occurrence of "the King of France" is primary, and true if it is secondary" (Russell, 1905, p. 490). In the narrow (secondary) scope, (a)"The King of France is bald" means that "x is bald" is asserted of the King of France, which is false, but this model does not exclude the possibility that it is true. One could say this on the grounds of incompatibility with the assertion (+a)"x is not bald" for the King of France. One can model the "incompatibility between a and +a", and therefore there is a extension in which the falsehood of the first proposition may be understood. In the wide domain or primary domain, the proposition "The King of France is bald" is asserted about x, which can only be false because the rule projected by the proposition cannot represent any instance; there is no instance of the King of France with or without baldness. In this dimension, there is no (extensional) basis for the proposition "The King of France is bald" as opposed to "The King of France is not bald" because the scope of the proposition is broad and does not allow us to specify a category of instantiation that would give the two propositions distinctness. One can not model that distinctness nor code its incompatibility. In this analysis, they remain indistinguishable. From the point of view of first order extensional logic, they are – curiously – the same proposition (they are confirmed and refuted by the same instances).

In order to avoid conflicts between the truth tables that govern interpretation, Russell sacrificed something: the law of unitary identity of the proposition. Some sentences have secondary analysis. Propositions that are not extensional functions of their constituents could only be stated in another sense, for example, as an assumption about the "sense" (Frege) or (in Russell's case) about the general x that falls within the scope of a complex and heterogeneous predicate (whose instances are the general x that satisfies the condition of being the "bald kings of the country called France").

This brings us to the origin of the problem that led to Kripke's objection. Frege's solution promised to explain intensional content, and Russell's solution teaches how to remove intensional content from logical analysis and translate it into an extension of a propositional function. Both teach how to block Leibniz's law of identity for problematic cases and reconcile scientific semantics with unavoidable anomalies, albeit harmless from this perspective. This was the price paid by the classical logicians. Certain prices seem large, out of season. But the principle of compositionality was very promising in Frege-Russell's time. It answered several problems about the nature of language and about the cumulative nature of the predicate "is true". Giving up a unified reading of the propositional content of Leibniz's law was a small price to pay compared to the promise of the new classical logic.

Interestingly, Kripke's critique suggests that this price was not worth it. The classical solution seemed to Kripke to be an enormous and unreasonable effort. Leibniz's law of identity should not be regarded as an obstacle needing exceptions. The propositional problem should always be approached and solved together when it comes to resolving the question of the truth or falsity of the sentence that expresses it. And this, Kripke argues, includes the problem of what is true of the proposition even under counterfactual circumstances. If the proposition does not have a unitary interpretation and we cannot establish that by fixing the reference in alternative scenarios, it makes no sense for him to say that we know to *model* the propositional problem (to select what would be *necessary* for the sentence to be true). Developing exceptions to the law of identity to account for modal and intensional contexts only circumvents the problem, which is always to select the worlds in which the proposition *cannot* be false.

3 RUSSELL AND THE EXTENSIONALIZATION OF NECESSITY: THE FALIBLE ASPECT OF NECESSITY IN PROPOSITIONAL FUNCTIONS

In this article we argue that the exceptions to Leibniz's law of identity studied by Frege and Russell do not "avoid the propositional problem." They provide a solution to problems of propositional determination that nevertheless involve resources of classical logic and do not invoke fixed essential or transmundane entities by rigid identifiers. It is an empiricist-fallibilist oriented solution. It retains the advantages of empiricist-fallibilism against essentialism. The problem does, however, involve algebraic solutions for determining identities. In this chapter we will go into more detail about Russell's famous solution: his theory of propositional functions.

Russell's thesis on propositional functions has suffered from the systematic charge of obscurity. More than that, it has been faulted for failing to accomplish the task it set itself. The work to which we refer is intended to contribute to the ontological depopulation of mathematics and semantics by proposing propositional functions as replacements for classes and terms in set theory which propel possibilities of paradox, one of which is known today under Russell's own name. In what follows, we cite two authors who are consistent with this view, namely Quine and Soames:

Russell [...] had a no-class theory. Notations purporting to refer to classes were so defined, in context, that all such references would disappear on expansion. This result was hailed by some [...] as freeing mathematics from platonism, as reconciling mathematics with an exclusively concrete ontology. But this interpretation is wrong. Russell's method eliminates classes, but only by appeal to another realm of equally abstract or universal entities—so-called propositional functions. The phrase 'propositional function' is used ambiguously in *Principia Mathematica*; sometimes it means an open sentence, and sometimes it means an attribute. [...] Such reduction comes to seem pretty idle when we reflect that the underlying theory of attributes itself might better have been interpreted as a theory of classes all along [...] (Quine, 1980, p. 122–23).

And Soames (2008, p. 217):

[Russell] speaks confusingly and inconsistently about [propositional functions], and the view that seems to be uppermost in his mind—that they are expressions—is obviously inadequate. Although other choices— extensional¹ functions and gappy propositions—make more sense, he doesn't systematically explore them, and [...] they aren't promising candidates for achieving ontological economies anyway.

Our strategy in this article is not to intervene directly in this controversy. In order to depart from it, we will use a premise as a starting point. On this premise, Russell's concept of propositional functions finds its utility somewhere between the reductionism of logical positivism and his extensionalism. We hold that propositional functions are strategies for making explicit the extensional knowledge of modal and intensional propositions that project complex identity concepts or depend on second-order quantification. Thus, they are instruments of extensionalizing discourse. To cite an author who we believe is consistent with our view, we can cite Ruth Barcan Marcus (1961, p. 311). The author, considering the problem created by (10) The evening star eq the morning star and (15) Scott is the author of Waverley, says:

One solution is Russell's, whose analysis provides a translation of (10) and (15) such that the truth of (10) and (15) does not commit us to the logical truth of (10) and (15), and certainly not to taking the 'eq' of (10) as identity, except on the explicit assumption of an extensionalizing axiom. Other and related solutions are in terms of membership in a non-empty unit class, or applicability of a unit attribute. But whatever the choice of a solution, it will have to be one which permits intersubstitutability, or some analogue of intersubstitutability for the members of the pairs: 'Scott' and 'the author of Waverley', and 'the evening star' and 'the morning star', which is short of being universal.

What the theory of propositional functions proposes is a way of analyzing sentences in which complex and higher-order identifiers appear, so that the analyzed sentence does not mention or refer to them. Extensionalized sentences emerge instead. An extensionalized sentence is one in which the problem of identity of terms is encoded without problematizing the necessity or intension of the identity. Marcus calls this extensionalization a weakening of the notion of identity, and understands it as transforming the problem of identification between Scott and Waverley, or the morning and evening stars, into a problem of coextension of their instances. For she, this solution is not universal. We do not

agree with this. Russell, of course, does not condone solutions that involve the universalization of an empirical identity. He does, however, have a theory about the universal learning of this identity, i.e., a theory about the logical content that is learned even with a purely empirical knowledge of the identity. Since this excludes the universality of the possible idealization of the identifying content, we can call this attenuation an empiricist move on the part of the author.

However, Russell did not appeal to logical empiricism, even if we accept his empiricism. His theory of eliminative paraphrases, or propositional functions, accomplished more than a mere analysis of empirical identities. Obviously, this leads to a new problem: how to describe the relational structure of identities whose similarity is not merely conceptual-logical, but often involves space-time or modal divergences. For Russell, we will argue, it comes down to figuring out what can be logically learned in a non-logical characterization of identity, i. e., a modal or intensional characterization of identity.

A propositional function is constructed in Russell's theory as a pillar in the development of his suspicion that certain symbols are incomplete symbolic fictions used in vulgar grammar to compensate for the absence of a definite *coordinate* (or the absence of a reference at all, as in the case of "The King of France"). An incomplete symbol like "The world champion of soccer" programs a semantic mapping that produces different values in different sentences. The ambiguity of the sentence can be unified, but not by a simple rule that programs the exclusion of "non-world champion". The simplest way to unify this ambiguity across all possible world champions is to create a higher order rule - a quantification over functional predicates. Sentences of the form "o φ ", or "(ιx) φx ", are defined in context:

$$A (\iota x) \varphi x = \text{df } \exists y[\forall x(\varphi x \leftrightarrow x = y) \wedge A(y)]$$

What is special about this reformulation is that it develops a superstructure of correlations between referential subconditions, in a kind of remapping of the sentence to φ . Remapping allows us to map the extension of equivalences between $x \varphi$ and $x \iota$, which act as supercompatibilities or type/category compatibilities. The basics for finding structural morphisms that allow this correlation-assignment can be found in the tools of modern algebra. Russell's

intuition is that it is possible to transform this kind of second-order compatibility between different "things" (the team of Germany, Brazil, France, etc.) that are (at a given time) world champions into regular and predictable information in propositional combinations, provided that we can reformulate the sentence in which the description - or the class, attribute, etc. - occur into a sentence in which it does not occur, and variables associated with quantifiers occur in its place. It is no accident that Quine and Soames recognized the limited utility of this solution to depart from the discourse on classes. In the end, a notion as obscure as that of correlation or supercompatibility will take the place of the notion of class. It hardly matters what kind of mathematical strategy is used if we are forced to admit rules for superstructural correlations that can only be systematized at a categorical or second-order level. If we judge Russell's solution in this way, the ability not to talk about classes is the least of its merits. It has other merits though.

But then what do we get from the theory of propositional functions? We argue in this article that the formulation of eliminative paraphrases supplemented by functions is an asset for systematizing the knowledge we have about provisional reference conditions or identity encodings restricted to context. The true brilliance of the theory of propositional functions, then, can be gaged by how it facilitates the determination of the difference between truth and falsity in problematic - modal, hypothetical - assertion contexts. If the law of excluded middle is to be useful in a theory of evidence, what is true must be false in only one way, even under counterfactual conditions (possibilities taken as actually given). To unify the ways in which the mapping of truth to "The world champion has 11 players" is not false, we must be able to include the extension of this proposition in the extension of the predicate "is false" when we negate it. However, this would not be possible if "the world champion" were involved in the meaning of the sentence as a "complex substance," because in that case it would make no sense to ask the question, "What would the world be like if the world champion did not have 11 players?" This would be a pseudo-problem if the descriptions were part of the proposition, because then identity would never be problematic: it would always be contradictory to deny it. If we did not have the ability to stabilize our notion of reference by omitting descriptions and replacing them with quantified variables, we would also lose our ability to idealize conditions or build models where the proposition would be false. We would be trapped in a

fraudulent propositional knowledge, in the sense of fraud that we associate with deception: The truth possibilities of the proposition would be determined without consideration of the possibility of falsity. It is in this context that the value of propositional functions becomes apparent. If we transform this sentence into another in which certain descriptions disappear and bound variables occur in their place, we construct a model for the anti-extension of the predicate "is true" even in non-referential or ambiguous circumstances. Russell thus restores the conditions of bipolarity for hypothetical or non-referential propositions, anticipating solutions that, under conditions of competitive fairness, compete with post-Strawsonian solutions that presuppose the meaning failure of non-referential propositions and postulate the value "neither-true-nor-false" in trivalued logic.

A propositional function is a translation of a hypothetical condition into a categorial-assertive condition that pays that price: the formulation of the verification rule or possible fulfillment as a kind of password or sign that acts as a key to different values in different contexts. From here we can evaluate a subthesis of Russell's, namely that modal predicates and existential ascriptions are properties of propositional functions. This thesis assumes that human speakers have been using complex second-order resources to make references to their propositions for much longer than we are aware of. When someone says that 'it is possible that Chile is world champion', he reserves a value for a propositional function, i.e., they develop a kind of password to increase the extension of "Chile is world champion" in antagonistic relation to the extension of "[...] it is impossible". Since we are able to generate assertive-categorial sentences with hypothetical conditions, these sentences are semantically formulated as a kind of dormant codes that are activated only under certain circumstances to unlock a semantic value. The semantic value of the problematic sentence is not a reference or an instance. It is nothing but the knowledge of a class of possible propositions or a function of propositions. For terminological precision, we might say that knowledge of a propositional function is pre-semantic or the mere knowledge of a partial contribution to a semantic value.

This way seems to lead to a disparaging view of modal sentences, as if they do not have the right to have an independent content. This may lead the reader to the typical thought that Russell belonged to the dominant crowd in the school of thought of his time: the connection between analytic philosophy and

logical positivism. Thus, he would have classified modal propositions with the same rigour with which other analytic and positivist philosophers judged the potential significance of ethical and aesthetic propositions. To dispel this impression, we agree with Dejnoška. According to him, Russell:

[...] found modality important enough to analyze it in at least semi-formal terms, and his analysis implies a modal logic (actually several). His approach is economical, even elegant: he eliminates and formalizes possibility in the same way that he eliminates and formalizes existence (Dejnoška, 2015, p. 2-3).

Russell's reduction and formalisation of modality provides us with an interesting explanation or modelling strategy for our thinking about modality that is compatible with empiricism – as opposed to essentialism. It is thus compatible with an orientation that respects the experience in the investigation of possible truth; and it does not presuppose the existence of the *possible* as relations between abstract objects or possible worlds. It is an explanation of the kind of reasoning that is present when we use sentences with modal predicates in an empiricist orientation. This choice of ways reduces the truth value of modal propositions to a pre-value, so that all that remains of propositions such as "Aristotle could have been Plato's teacher" is a kind of ingredient value that is still incomplete and can be defined according to its explanatory contribution to a proposition about "possible aristotle" (Aristotle in possible worlds).

On the basis of Russell's argument, it is clear that the possibility of encoding the extensional compatibility between "being bald" and "being the king of France" exists only in a restricted or secondary framework. We are given only the possibility of assigning different values to the sentences "The king of France is bald" and "The king of France is not bald" in the semantic analysis programmed to search for a "possible instance" or counterfactual. The difference between these two sentences is not semantically determinable in the other analysis. That is, it is not a difference that affects the truth or falsity of the sentence, but at best a non-semantic difference. This means that Russell's answer to Kripke would be that he actually has a way of reading the "counterfactual" content of the proposition. But it is a fallible way, instead of an essentialist one.

4 A POSSIBLE ANSWER FROM KRIPKE

We find in this article a sense in which the divergence between Frege-Russell and Kripke is defined as a broad divergence about the nature of identity. Both Frege's antipsychologism and Russell's empiricist phase tended toward the idea that identity is a logical-mathematical construction rather than a metaphysical-psychological fact. If Russell is right, then the price of expanding the extensional universe to check sentences with non-assignable content - such as The King of France does not exist - is that we establish an algebraic rule in which we talk about everything that can instantiate a variable. In this way, the non-assignment of x to King of France is mapped to a superextension that converges to the antiextension of the predicate "x exists". The structural properties of this identity can be described algebraically. Widescopists would say that producing codes for locating counterfactual conditions amounts to *extensionalizing* predicates such as "it would be possible", and for that we do not need strong notions of identity. Kripke can only reply that the deep nature of the problems is not the same. These problems – the extensionalization of “would be” and the reference in possible worlds – coincide only on the surface. The rigidity problem aims at determining who the King of France is under potential compatibility conditions, while the extensional problem aims at something more general: mapping an algebraic supervalue onto propositions in which the description "The King of France" occurs in order to identify it in valid equations. For Kripke, this looks like an artificial mathematical solution that does not locate what the sentence wants to map, namely the possible semantic value of the sentence.

The quote on page 9 of Naming and Necessity suggests that Kripke is uncomfortable with the notion that compatibility between algebraic variables in second-order equations represents the same knowledge we have about identity between two names:

[...] my objection to Russell is that all the many propositions expressed by various readings of (I) (assuming that in all readings 'Aristotle' is a proper name) would, if he were right, fail to conform to the rule of rigidity. That is, no such proposition conforms to the rule that there is a single individual and a single property such that, with respect to

every counterfactual situation, the truth conditions of the proposition are the possession of the property by that individual, in that situation (Kripke, 2001, p. 9-10).

What we know when we know that Aristotle is Aristotle seems to Kripke to be different from what we know when we work out algebraic rules for encoding the identity of a description and a name or two descriptions for Aristotle. The author seems skeptical about the potential of Algebra to give us knowledge about real, substantial, or causal identity. This skepticism is rational: the last time identity was seen as mere numerical resemblance, the intellectual universe experienced the greatest skepticism about the idea of cause and substance. Hume, not unlike Leibniz, doubted that mathematical and numerical correlations could provide a sufficient reason why things should be one way and not another. Mathematical identity, then, gives limited knowledge of substance. At most, this compatibility expresses numerical indistinguishability between two classes, but that would not give us the kind of metaphysical knowledge about what makes a thing remain the same in other worlds.

Kripke, who values Leibniz's law more than extensionalism, thinks that abandoning the latter is less harmful than the former. For him, Leibniz's law does not fail under any circumstances. There are no exceptions to the law of identity. Rather, there are certain epistemic circumstances that make it appear that this law fails – just as Leibniz believed that we confuse identical things with distinct things because we do not possess the intelligence of God. In a metaphysical reading (and in the mind of God), the law never fails. The confusion, then, results from the inability to distinguish between epistemic and metaphysical problems (although the author does not take Leibniz's step and define metaphysical possibility as coextensive with God's knowledge).

5 NON-SEMANTIC NECESSITIES AND RUSSELL'S NON-SEMANTIC SOLUTION TO THE PROBLEM OF DESCRIBING THE EXTENSION OF "POSSIBILITY"

As we have seen, Kripke takes the position that the logic of propositional construction has a dimension of counterfactual identification that goes beyond mere truth-functional extensionality and weak extensional identity. Thus, he

concedes the importance of necessary but not *a priori* propositions, that is, propositions that we recognize as necessary through strategies of reference setting. Since these strategies are concerned with establishing values for hypothetical conditions, their meaning is not extensional, and they do not establish a generalization rule for the sentence's contribution to semantic compositions. It does justice to the way empirical science organizes the study of counterfactual cases by testing propositional problems about the strength of assertion to possible cases. It shows how our semantic devices enable stable representations of propositions with high generality, and how we are able to semantically represent the divergence between necessary propositions by recognizing speculative conditions. But this solution raises more challenges than explanations in terms of how compositional semantics can justify the construction of meaning from the building blocks of its structure. For language responds to intensional identity conditions in non-compositional ways: It maps values to intensions, but at the cost of adding superstructures that increase the number of contingent things that would have to be learned to form sentences from the basic grammar. Therefore, it is not the best option if we want to make the results of the common science practitioner compatible with realistic semantic interpretations, that is, with recognizable semantic interpretations of the dispositional and necessary expressions of science. It does not show what transformations take place in language, what compositional changes and structural modifications are necessary to represent a revision of necessary expressions, or to show a scientifically or culturally obsolete identity - a synonym or a posteriori logical identity - as fallible.

Kripke's traditional assessment of the problem can be revised. One could reverse the positions and place Russell later on the timeline of philosophy. The conflict would then be seen as a struggle between an essentialist and an empiricist mentality. Part of the success of Kripke's theory is due to the veiled rejection or gradual decline in acceptance of two theses: Extensionalism and Empiricism. However, as long as the latter two theses are alive and well as competing philosophical programs, the "losing side" will still have some voice. Had Russell come late in the timeline, his thesis would probably chide Kripke precisely for trying to provide a unified view of the law of identity, i.e., a version of Leibniz's law that also applies to relative conditions, empirical necessities, and

contingent propositions. Ironically, this is precisely what Kripke sees as the advantage of his view. An empiricist would perceive it as representing a purely algebraic knowledge of possible solutions to equations (the knowledge of a propositional function) as if it were a metaphysical knowledge of identity between substances. One wonders how Kripke would explain the fallibility of counterfactual identities determined by scientific theories that develop better laws than the laws of previous scientific theories. What kind of knowledge would constitute this subversion and scientific revolution? This difficulty does not decide this conflict, but at least suggests that empiricism – in its post-Quine variants – are still strong contenders in the philosophical marketplace, since they can account for the revision of logical identities simply by rejecting strong modalities of identities, such as necessary identities.

Russell's theory also does not touch the problem of transforming language to accommodate new modal or dispositional knowledge. But his theory at least has the advantage of showing that knowledge of some *a posteriori* necessities - or some absurdities that cannot be detected by mere logical contradiction - corresponds to knowledge of some sentence structures when properly analyzed as fallible logical necessities; i.e. the knowledge of a necessary propositional function which is regarded as false. The knowledge generated by this functional extensionalization of complex identities is not semantic, for it does not teach how to determine the structure of sentence composition. But Bertrand Russell was not a semanticist. His interest was in providing mechanical and mathematical tools to make consistent mappings to scientific, modal, hypothetical propositions, and to dangerous predicates such as "exists."

Russell's view is that when he examines how propositional analysis behaves under negation, i.e., how it represents the fallible character of some complex identities, he is working on a problem of determining the extensional content of predicates such as possibility and necessity. For Russell, these predicates are nothing more than the rules we use to give artificial or fictional extensions to statements that need stable meaning, even if not true or false, like hypotheses. However, this extension is not like a normal instance or a group of instances. It is the equality of a group of propositions described by their relational structure. It provides an algebraic representation of "possible truth". It constructs the anti-extension of the predicate "is true" under counterfactual conditions.

For the English author, the necessity of removing the existential content from the predicate "exists" was on the same list of requirements as removing the causal content from the theoretical connections and removing the factual content from the necessary logical connections:

Unfortunately, however, the definition in Baldwin's Dictionary says that what is necessary is not only "true under all circumstances" but is also "true." Now these two are incompatible. Only propositions can be "true," and only propositional functions can be "true under all circumstances" (Russell, 1917, p. 182).

In a sense, when someone constructs a propositional function, they are working out the fallibility of logical truth statements, i.e., they are representing what it means to use expressions like "impossible" in a defined semantic compositional context, even though it is a context of dispute and controversy in which there is theoretical and rational disagreement about what is possible. He is stabilizing knowledge about the "possible" in a decidable environment. Propositional functions provide a stable, but not infallible, interpretation of impossibilities. It shows that denying necessary propositions in higher speculative contexts is a feasible action. This is nothing more than knowledge that provides the ability to investigate, confront, and discuss counterfactual conditions in semantically recognizable ways, i.e., ways that can be adjusted to the principle of semantic compositionality and extensionality. For Russell, this understanding brings about a kind of learnable logical knowledge that can retain its extensional character even going beyond current paradigms of meaning – i.e., an extensional knowledge not dogmatically fixed by semantics. And this is what matters: a logical knowledge that is up to the tasks invoked by Kripke and the "rigidity problem" – but without the semantic shortcomings that arise from violating the extensionality principle.

6 FINAL CONSIDERATIONS

Our conclusion can be formulated as follows. First, Russell was never really indifferent to the counterfactual problem, and the same could be said of Frege, especially if we consider Sinn's notion as his way of discussing the same issue. The same issue under discussion is how to evaluate possible ways of

violating Leibniz's law of identity without losing the ability to know the result semantically through some rule, in Frege's case through intensional knowledge, in Russell's case through knowledge of a propositional function. What matters is the kind of knowledge one has when speaking in a non-extensional way, that is, when locating this identity not referentially but as a sense or identity between descriptions understood as a linguistic content.

Russell, however, goes beyond Frege in developing functional ways to recover extensionality under these conditions by showing that knowing these rules - a propositional function - is equivalent to knowing how to falsify these identity statements, i.e., how to construe the model to describe the extension in which modal or counterfactual identities are false. In this way, one can think about the conditions of logical learning that are incompatible with semantic learning. Since language emerges only after empirical discoveries, we cannot expect languages to be able to semantically express the necessity of "men are mammals" before scientific inquiry encodes the structural relation in which that identity exists. There are fallible logical truths that a language cannot express in a way consistent with a comprehensive theory of meaning that aims to predict their meaningful statements. They are semantically unpredictable, but they can be part of an empirical investigation in which possible extensions are postulated even before we have semantically unambiguous sentences to express them by straightforward formal mappings. Thus, existential sentences without a mappable reference, such as "The King of France does not exist," can be evaluated as true or false insofar as the anti-extension of the predicate "exists" is determined by a propositional function.

With Frege, modality and *intensional identity* emerge in our discourse as a step beyond logical truth. In Russell, modality and *superextension* emerge in our discourse as a step beyond logical truth. Both provide resources for understanding logical truth under conditions where we are able to prove it false, or revise it. The assumption behind these theories is that we use expressions like "it is necessary" reasonably only under two conditions. First, we do so when the logical learning associated with that knowledge is already theoretical, problematic, fallible; and second, when that fallibility can be expressed intensionally or, when it remains extensional, it is expressed simply as the

extension of a propositional function (a super-extension), like the "false" rows in a truth model for a propositional function.

Thus (in both cases) identities that are not connected by the law of non-contradiction, such as epistemic identities, can be distinguished, not semantically (because the truth or falsehood holds only for a higher-order knowledge that has no singular reference) – but as a ingredient knowledge (Dummett). They are part of our knowledge of the categorical relations that favor identity, as opposed to the conditions - possible worlds - that would abrogate identity. Learning how to apply a modal term, then, is a knowledge of the possible ways in which an assertion of identity can be contested. In other words, it is knowledge about how to generate the anti-extensional interpretation of the predicate "true" under counterfactual verification conditions.

Learning how to apply a modal term, then, is a knowledge of the possible ways in which an assertion of identity can be contested or rationally discussed. Rational discussion comes before semantic expression: we can rationally discuss the non-existence of the King of France even if there is no semantic functional composition that expresses the predicate "non-existent" extensionally. We can extensionalize complex predicates in a rational discussion even if our language does not help us to do so - that was, after all, the task of logical analysis in the early days of analytic philosophy. In other words, Russell teaches how to gain knowledge about how to generate the anti-extensional interpretation of the predicate "true" under counterfactual verification conditions. One could say that it is sub-semantic knowledge, in the sense that it is on its way to becoming fully semantic, since its potential truth can be fully distinguished from potential falsity at this moment of empirical inquiry - and therefore at this point of empirical inquiry there are semantic decision experiments (models) to determine the meaning of it.

This sub-semantic knowledge shows what we "learn" differently from two equally false or unconfirmed theories. We learn more - our working knowledge is more complete - when we construct a theory about truth whose extension is antagonistic to a false or unconfirmed scientific theory than when that theory about truth has an antagonistic extension to superstition or religious dogma. We learn nothing about truth by knowing the anti-extension of "brides in ships bring misfortune." But we can greatly enrich our knowledge of truth if we create

consistent models to describe what would be the case if amoebae were more evolved than mammals, that is, if we can imagine with logical rigor what would be the case if the theory of evolution (without adaptations) were false.

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